Towards non-invasive electrocardiographic imaging using regularized neural networks

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ABSTRACT

We present a new data-driven technique for non-invasive electronic imaging of cardiovascular tissues using routinely-measured body-surface electrocardiogram (ECG) signals. While traditional ECG imaging and 3D reconstruction algorithms typically rely on a combination of linear Fourier theory, geometric and parametric modeling, and invasive measurements via catheters, we show in this work that it is possible to learn the complicated inverse map, from body-surface potentials to epicardial or endocardial potentials, by exploiting the powerful approximation properties of neural networks. The key contribution here is a formulation of the inverse problem that allows historical data to be leveraged as ground-truth for training the inverse operator. We provide some initial experiments, and outline a path for extending this technique for real-time and cross-patient diagnostics.

THE FORWARD MODEL

We begin by modeling the forward electromagnetic problem with time-dependent Maxwell's Equations, as:

$$\begin{aligned} \nabla \cdot B &= 0 \\ \nabla \times H - \frac{dD}{dt} &= J \\ \nabla \cdot D &= \rho_v \\ \nabla \times E + \frac{dB}{dt} &= 0 \end{aligned}$$

where ρ_v and J respectively represent models of charge and current distributions in the myocardium, and E and B represent the resulting electric and magnetic fields throughout the torso and on the surface of the body.

If we assume that the torso can be modeled as linear dielectric media (i.e. $D = \epsilon E$ and $B = \mu H$), these equations simplify to a system with two variables.

That is, if we make these substitutions, apply a few vector identities, and assume we can lump the source-terms into a single forcing term of compact support, this system reduces exactly to the inhomogeneous Helmholtz equation:

$$\nabla^2 A + k^2 A = f(x)\,,$$

where we note that the equation is identical for $\mathrm{A} = \mathrm{E}$ and $\mathrm{A} = \mathrm{B}$ (i.e. the electric and magnetic field are uncoupled).

The Helmholtz equation has analytic solutions, commonly formulated using spherical Bessel functions, and is the basis of classical Fourier imaging theory. With a suitable choice of boundary condition (e.g. Sommerfeld), this equation admits solutions, $A(x)$, given by the convolution:

$$A(x) = (G \star f)(x) = \int_\Omega G(x-y) f(y) \delta y\,,$$

where in 3D problems in a single medium, the Green's function, $G(x)$, is:

$$G(x) = \frac{e^{j \cdot k \cdot |x|}}{4\pi |x|}$$

which can be understood as a spherically symmetric wave.

THE INVERSE MODEL

In many cases, the Green's function formulation of the inhomogeneous Helmholtz equation affords a linear algebraic structure ($Gx = b$), whose least-squares solution can be written down as:

$$\hat{x} \approx (G^H G)^{-1} G^H b\,,$$

where in this case G is the Green's kernel constructed by incorporating the torso and cardiac electrode geometry, b is the measured wavefield on the torso, and x is the unknown source-wavefield or cardiac potentials to be predicted. However, in practice there are many numerical issues with computing this quantity directly. Instead, we assume the existence of some pseudo-inverse operator $G^\dagger$ and allow these parameters to be free variables which we solve for in the "training" phase of our algorithm. For example, this can be done by introducing a new term L, as:

$$\hat{x} \approx L\, G^H\, b$$

In the imaging literature, this reconstruction algorithm is known as filtered back-projection (FBP), and is a popular tool because it affords a simple and fast solution to the inverse problem. However, at modest sampling frequencies ($\leq$ 10kHz) and dielectrics $\epsilon_r \leq 100$ the Green's kernel G is nearly a uniform matrix with extremely low spatial bandwidth. Thus, directly applying G^H diffuses the measured wavefield data, producing very poor spatial resolution in the reconstruction. In fact, this has historically been the limiting factor in low-frequency imaging, such as ECG, with theoretical results requiring operating frequencies orders of magnitude higher than what is typically measured.

In this paper we argue that, through the systematic introduction of multiple algebraic terms, we can turn the reconstruction and estimation task into a non-linear problem that can overcome some of the limitations posed by linear Fourier theory. For example, in our experiments, we found the following network structure outperforms the simple 2-layer structure of FBP on the sample dataset used:

$$\begin{aligned} a_1 &= G^H \cdot b \\ a_2 &= L \cdot a_1 \\ a_3 &= a_2 + c \\ a_4 &= w_0 + w_1 a_3 + w_2 a_3^2 + w_3 a_3^3 \end{aligned}$$

where the newly introduced variables are $c, w_0, w_1, w_2, w_3 \in R^n$, and the for the k-th entry of $\hat{x}$ we may write:

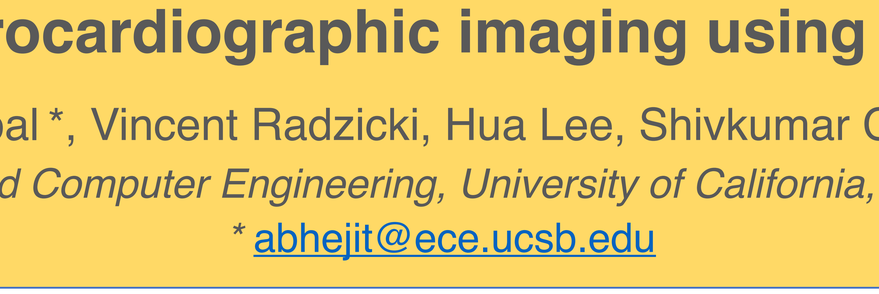
$$\hat{x}_k = \sum_{p=1}^{2} \omega_{p,k} \left(\left(\sum\nolimits_j^n \sum\nolimits_i^m \left(L_{k,j} \cdot G_{j,i} \right) \cdot b_i \right) + c_k \right)^p$$

which defines the computation required for reconstruction or prediction of the potential at a spatial point corresponding to the k-th electrode.

RECONSTRUCTION RESULTS

To demonstrate the feasibility of our approach, we used an open-source dataset made available by the Consortium for ECG Imaging (CEI). The dataset was collected at Valencia Polytechnic University, and involved two clinical examples of human atrial fibrillation mapping with body surface mapping.

In our initial experiments, we trained our physics-inspired neural network to predict potentials on the epicardium from body-surface potentials, using a simple 2-norm error with a Sobolev-norm regularization or penalty term. The results are summarized in Table 1 (right), and Figure 2 (below).



(a) Ground Truth Spatial Map Epicardial Potentials (b) Reconstructed Spatial Map of Epicardial Potentials

Figure 1: Spatial Reconstruction of Epicardial Potentials

	Mean Error [mV]	% Error	σ [mV]
Training	0.310	12.07%	0.321
Testing	0.312	12.59%	0.321

Table 1: Reconstruction Performance

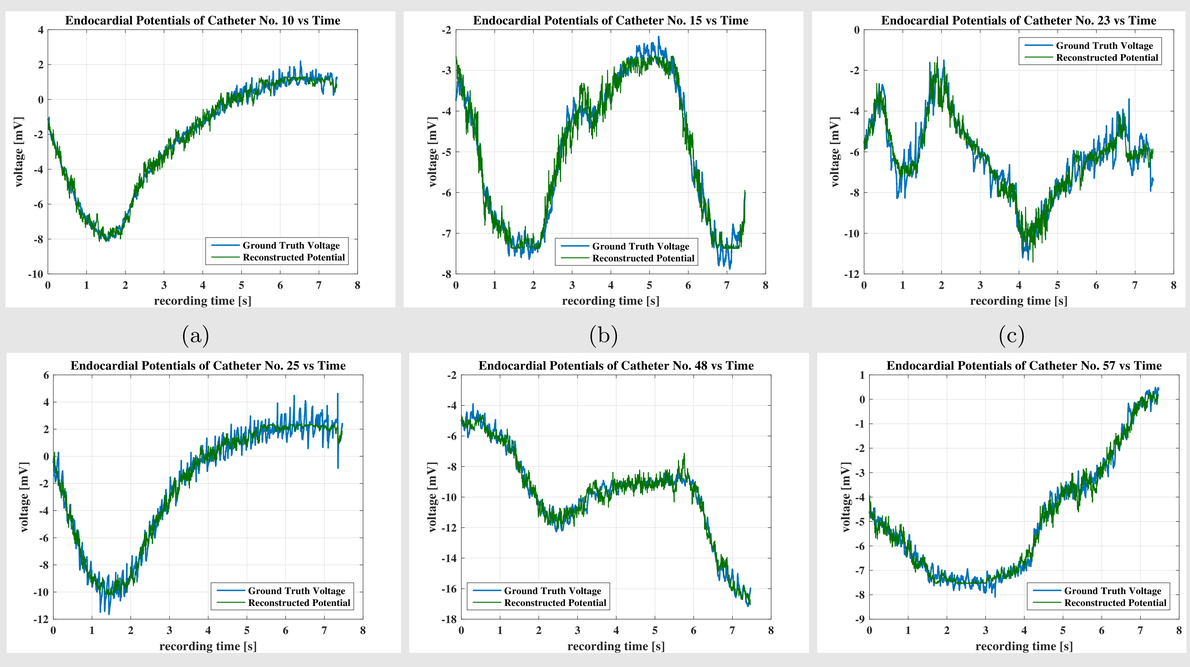
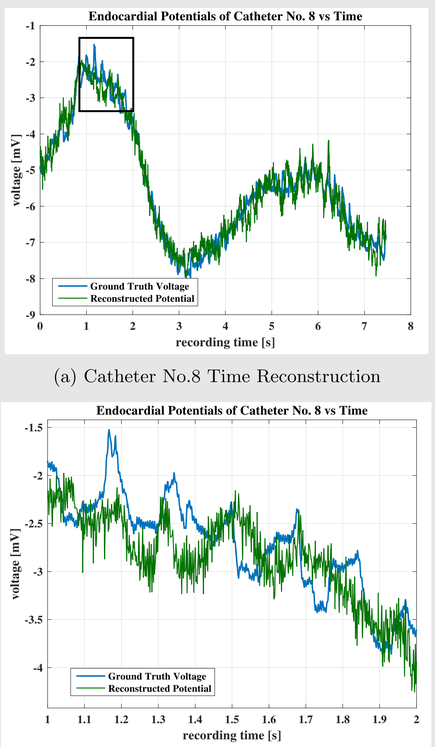


Figure 2: Time Reconstruction of Various Catheter Measurements

DISCUSSION

Although the reconstructed potentials closely track the ground-truth potentials both visually and quantitatively (with low *relative*-error), we believe there remains much work to be done to improve the use neural networks to complete the inverse map. In particular, if we examine the *small*-signal behavior, we find there to be a slight mismatch between the ground-truth potentials and reconstructions when viewed as a time-graph (Figure 3), indicating time-dependence in the data.



(a) Catheter No.8 Time Reconstruction

(b) Catheter No.8 Time Reconstruction Zoomed

Figure 3: Time-Reconstruction of Epicardial Potential Signal, (a) Large Signal, (b) Small-Signal

That is, in these experiments, the endocardial potentials were reconstructed or predicted from only body-surface measurements at every instant in time. As such, no time-frequency information was exploited in the neural-network algorithm, although this is typically how FBP is formulated. We believe such time-harmonic formulations will allow the neural network to learn to integrate over the relevant time-constants, smoothening the reconstruction.

CONCLUSION

We have demonstrated the feasibility of a data-driven approach for the non-invasive reconstruction and electronic imaging of human tissues using neural networks, by showing that it is possible to reconstruct interior epicardial potentials from body-surface ECG potentials to a high relative accuracy **(87%+)** when the geometry is fixed or known. We believe our results show promise in the use of non-linear formulations of the inverse problem that can practically overcome some of the limitations posed by the linear Fourier theory, even when trained or optimized using simple gradient-based methods. One interesting feature of our approach is that the structure and functional form of the presented approximation networks are modeled and initialized based on the linear theory, which we believe aids in the ability for the simple gradient-based optimization methods to find desirable local minima.